Observables vs. gauge symmetries, boundary observables and quasi-local holography

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Outline

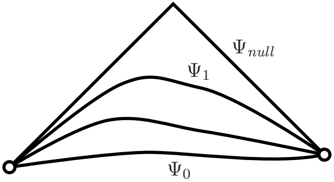
1. Introduction and Motivation
2. Subsystems and double null foliation
3. Why the Barbero – Immirzi parameter matters
4. Outlook and conclusion

Introduction and Motivation

Infra particles, coarse graining, edge modes, BH information

Why quantum gravity in causal regions? Different views:

- Mere gauge fixing: Represent diffeomorphism equivalence class of states $[\Psi_0]$ by states on the light cone.
- Coarse graining: Build observables by successively gluing gravitational subsystems.



- Soft modes/edge modes: In gravity, energy, momentum, angular momentum, center of mass, supertranslations ... are analogous to charge in QED. Do we have superpositions of such charges in nature? Can we study such charge superpositions in the lab? Help us understand black hole information loss?

[Strominger, Perry; Godazgar, Harlow, Wu; Prabhu, Chandrasekaran, Flanagan, Bonga; Bodendorfer, Thiemann, Thurn; Sahlmann; Krasnov, Ashtekar, Beetle, Krishnan, Corichi, Carlip, Giddings, Freidel, Donnelly, Speranza, Riello, Geiller, Livine, Dittrich, Pranzetti, Grumiller, Seraj, Barnich, Compère,...]

Gravity in terms of differential forms

To understand how gravity couples to boundaries, it is useful to work with differential forms rather than tensors since there is a natural notion of projection onto the boundary, namely the pull-back $\varphi^* : T^*M \to T^*(\partial M)$, which does not require a metric.

Fundamental configuration variables

$$\begin{aligned} &g_{ab} = \eta_{\alpha\beta} e^{\alpha}{}_{a} e^{\beta}{}_{b}, \\ &\nabla \wedge \omega^{\alpha} = \mathrm{d} \wedge \omega^{\alpha} + A^{\alpha}{}_{\beta} \wedge \omega^{\beta}. \end{aligned}$$

Palatini action

$$S[A, e] = \frac{1}{16\pi G} \int_{\mathscr{M}} *\underbrace{(e_{\alpha} \wedge e_{\beta})}_{\Sigma_{\alpha\beta}} \wedge F^{\alpha\beta}[A] + \text{boundery terms.}$$

Symplectic potential

$$\Theta_{\Sigma} = \frac{1}{16\pi G} \int_{\Sigma} *\Sigma_{\alpha\beta} \wedge \mathbb{d} A^{\alpha\beta} + \text{corner terms.}$$

Charges and symmetries 1/2

Two kinds of gauge symmetries: diffeomorphisms and internal Lorentz transformations.

Lorentz transformations

$$\delta_\Lambda[e^\alpha] = \Lambda^\alpha{}_\beta e^\beta, \qquad \Lambda_{\alpha\beta} = -\Lambda_{\beta\alpha}$$
$$\delta_\Lambda[A^\alpha{}_\beta] = -\nabla\Lambda^\alpha{}_\beta.$$

Lorentz charges are integrable at full non-perturbative level.

$$\Omega_\Sigma(\delta_\Lambda, \delta)\big|_{\mathrm{EOM}} = -\delta[Q_\Lambda].$$
$$Q_\Lambda[\Sigma] = -\frac{1}{16\pi G}\oint_{\partial\Sigma} *\Sigma_{\alpha\beta}\Lambda^{\alpha\beta}.$$

NB: Such Lorentz charges do not exist in metric gravity (on the ADM phase space). Physically meaningful perhaps only if we add fermions (defects of torsion).

[Freidel, Donnelly, Speranza, Riello, Geiller, Speziale, Paoli, Oliveri, ...]

Charges and symmetries 2/2

Two kinds of gauge symmetries: diffeomorphisms and internal Lorentz transformations.

Base diffeomorphisms lifted upwards into the Lorentz bundle

$$\begin{aligned}\delta_\xi[e^\alpha] &= \nabla(\xi\lrcorner e^\alpha) + \xi\lrcorner(\nabla\wedge e^\alpha),\\ \delta_\xi[A^\alpha{}_\beta] &= \xi\lrcorner F^\alpha{}_\beta\,.\end{aligned}$$

Diffeomorphism charges

$$\Omega_\Sigma(\delta_\xi,\delta)\big|_{\mathrm{EOM}} = \frac{1}{16\pi G}\oint_{\partial\Sigma}\xi\lrcorner *\Sigma_{\alpha\beta}\wedge\delta A^{\alpha\beta} \overset{?}{=} -\delta[P_\xi].$$

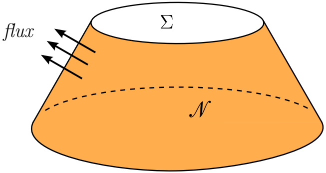
Trivially integrable at linear order in perturbations

$$\begin{aligned}e^\alpha &= \overset{\circ}{e}{}^\alpha + f^\alpha \equiv \overset{\circ}{e}{}^\alpha + f^\alpha{}_\beta\,\overset{\circ}{e}{}^\beta, \quad f_{\alpha\beta} = f_{\beta\alpha},\\ P_\xi &= \frac{1}{8\pi G}\oint_{\partial\Sigma}\xi\lrcorner *\overset{\circ}{\Sigma}_{\alpha\beta}\wedge\overset{\circ}{\nabla}{}^{[\alpha}f^{\beta]}.\end{aligned}$$

NB: for an asymptotic time translation $\xi^a = \left[\frac{\partial}{\partial x^0}\right]^a$, the linearised charge P_ξ returns the ADM mass for a linearised solution $f_{\alpha\beta} = \mathcal{O}(r^{-1})$ around $\overset{\circ}{e}{}^\alpha = \mathrm{d}x^\alpha$.

A puzzle: Integrability of charges

- In gravity, time evolution $t \to t + \varepsilon$ can be understood as a large gauge transformation.
- It seems reasonable to expect the Hamiltonian is the generator for such a gauge transformation:



$$H[\Sigma] \equiv P_\xi[\Sigma] \overset{?}{=} \oint_{\partial\Sigma} d^2v^a\, \xi^b T_{ab}[?].$$

- We assume that P_ξ generates the symmetry algebra

$$\{P_\xi, P_{\xi'}\} = -P_{[\xi,\xi']} + c[\xi,\xi'].$$

- However, that's at odds with the fact that a system may loose mass via gravitational radiation

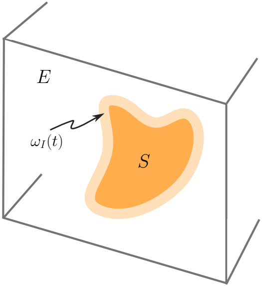
$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} Mc^2 = \frac{\mathrm{d}}{\mathrm{d}t} H &= \{H, H\} = 0, \\ &= -\frac{1}{4\pi G} \oint_{S^2_t} d^2\Omega\, |\dot{\sigma}^0|^2 \leq 0. \end{aligned}\right\} \quad \lightning$$

- ... unless, we allow for an explicit time dependence in the Hamiltonian ...

Space of histories vs. phase space

Trivial toy model: subsystem S that interacts with its environment E through some boundary degrees of freedom $\omega_I(t)$.

$$S[p_i, q^i|\omega_I] = \int_0^1 \mathrm{d}t \Big(p_i(t)\dot{q}^i(t) - H_o[p_i(t), q^i(t)] - H_{int}[p_i(t), q^i(t)|\omega_I(t)] \Big).$$



- Subsystem has symplectic two-form $\Omega_S = \mathbb{d}p_i \wedge\!\!\!\wedge \mathbb{d}q^i$
- Subsystem Hamiltonian is explicitly time-dependent $H = H_o[p_i, q^i] + H_{int}[p_i, q^i|\omega_I(t)]$

- Phase space: all trajectories $(p_i(t), q^i(t)) \ni \mathscr{P}_{\omega_I(t)}$ generated by the Hamiltonian flow for fixed $\omega_I(t)$).
- Space of physical histories: all possible configurations of bulk plus boundary fields $\bigsqcup_\omega \mathscr{P}_\omega = \mathscr{H}_{phys}$.

Space of histories vs. phase space

In GR, there is no preferred foliation and no preferred time variable t. The distinction between phase space, boundary fields and the space of physical histories becomes rather subtle [Harlow, Wu, Freidel, Pranzetti, Geiller, ww, Barnich, Compère, ...].

In this context, the covariant phase space approach [Ashtekar, Witten, Wald, Zoupas] allow us to infer the on-shell value of the Hamiltonian directly from the action.

For our simple toy model

$$S[p_i, q^i|\omega_I] = \int_0^1 \mathrm{d}t \Big(p_i(t)\dot{q}^i(t) - H_o[p_i(t), q^i(t)] - H_{int}[p_i(t), q^i(t)|\omega_I(t)] \Big).$$

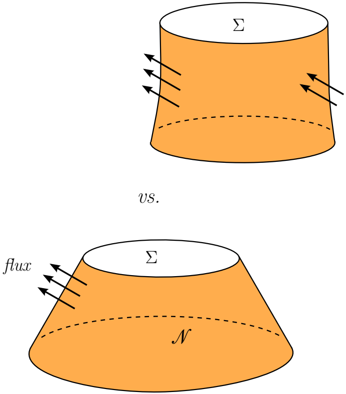
If $\delta \in T\mathscr{H}_{phys}$ is a tangent vector to the space of physical histories,

$$\delta[H]\big|_{\mathscr{H}_{phys}} = -\Omega_S\Big(\frac{\mathrm{d}}{\mathrm{d}t}, \delta\Big)\Big|_{\mathscr{H}_{phys}} + \frac{\partial H_{int}[p_i, q^i|\omega_I(t)]}{\partial \omega_I}\delta[\omega_I(t)]\Big|_{\mathscr{H}_{phys}}.$$

Subsystems as evolving regions in space

To characterise a gravitational subsystem,
two choices must be made.

- A choice must be made for how to extend the boundary of the partial Cauchy hypersurface Σ into a worldtube $\mathscr{N}$.
- A choice must be made for what is the flux of gravitational radiation across the worldtube of the boundary, i.e. a (background field, c-number) that drives the time-dependence of the Hamiltonian.

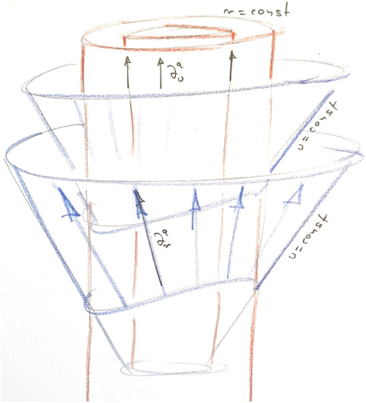


N.B.: In spacetime dimensions $d < 4$, there are no gravitational waves, and we can forget about the second issue. The Hamiltonian will be automatically conserved.

Double null foliation and quasi-local phase space

Bondi coordinates, Newman Penrose null tetrad

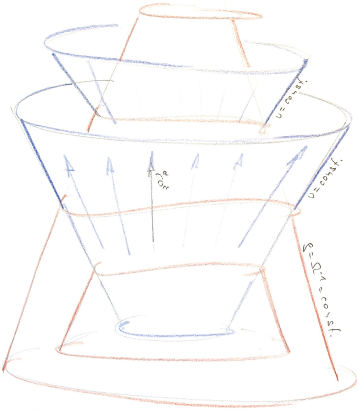
Most common gauge choices:



- Retarded time u and radial coordinate r.
- Vector field $k^a = \partial_r^a$ is null, and r is an affine parameter.
- Vector field k^a is surface forming, $k_a = -\nabla_a u$.
- Extend k^a into null tetrad $(k^a, \ell^a, m^a, \bar{m}^a)$, which is parallel propagated along the null rays k^a.
- NB: the dual null vectors ℓ^a are *not* surface orthogonal.
- This is disadvantageous for our purpose: we want to assign a phase space to finite regions bounded by *ingoing* null surfaces, then take the limit to null infinity afterwards.

Double null foliation

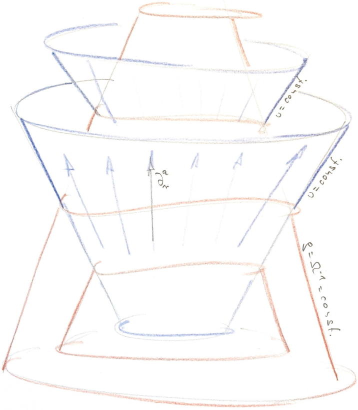
Instead, we use a double null foliation:



- Retarded time u and inverse conformal factor $\rho = \Omega^{-1}$ (advanced time).
- Vector field $k^a = \partial_r^a$ is null, and r is an affine parameter: $\rho = r + \mathscr{O}(r^{-1})$.
- Vector field k^a is surface orthogonal and $k_a = -\nabla_a u$.
- Extend k^a into null tetrad $(k^a, \ell^a, m^a, \bar{m}^a)$ such that vector field ℓ^a is surface orthogonal and $\ell_a \propto \nabla_a \rho$.
- $U(1)$ gauge condition $m_a k^b \nabla_b \bar{m}^a = 0$.
- This is advantageous for our purpose: for every ρ, we have a region $\mathscr{M}_\rho$ bounded by a null surface $\mathscr{N}_\rho$ (and two disks at bottom and top).

Falloff conditions, adapted Newman – Penrose spin dyad

Spin dyad (k_A, ℓ_A) and associated Newman – Penrose tetrad
$(\mathrm{i}k^A\bar{k}^{A'}, \mathrm{i}\ell^A\bar{\ell}^{A'}, \mathrm{i}\ell^A\bar{k}^{A'}, \mathrm{i}k^A\bar{\ell}^{A'}) = (k^a, \ell^a, m^a, \bar{m}^a)$.



Outgoing shear and expansion

$$\vartheta_{(k)} = q^{ab}\nabla_a k_b = \frac{2}{r} + \mathcal{O}(r^{-2}),$$

$$\sigma_{(k)} = m^a m^b \nabla_a k_b = \frac{\sigma^0(u,z,\bar{z})}{r^2} + \mathcal{O}(r^{-3}).$$

Ingoing shear, expansion and non-affinity

$$\vartheta_{(\ell)} = -\frac{\mathscr{R}[\mathscr{C}]}{2r} + \mathcal{O}(r^{-2}),$$

$$\sigma_{(\ell)} = m^a m^b \nabla_a \ell_b = -\frac{\dot{\sigma}^0(u,z,\bar{z})}{r} + \mathcal{O}(r^{-2}),$$

$$\kappa_{(\ell)} = \frac{\mathfrak{Re}(\Psi_2^{(0)})}{r^2} + \mathcal{O}(r^{-3}).$$

Peeling and Weyl spinor

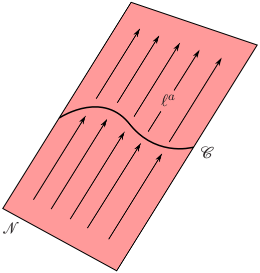
$$F_{AB} = \Psi_{ABCD}\Sigma^{CD}, \quad \Sigma_{AB} = \frac{1}{2} e_{(A}{}^{C'} \wedge e_{B)C'},$$

$$\Psi_s = \Psi_{A_1 \dots A_4} \ell^{A_1} \dots \ell^{A_s} k^{A_s+1} \dots k^{A_4} = \mathcal{O}(r^{s-5}).$$

Back to the covariant phase space

Self-dual two-forms on a null surface

On a null surface it is useful to work with forms rather than vectors. Given a tetrad e^α, we have a hierarchy of p-forms: $e^{\alpha_1}\wedge\dots\wedge e^{\alpha_p}$.



- Directed area two-form $\Sigma^{\alpha\beta} = e^\alpha\wedge e^\beta$

$$\begin{pmatrix}\Sigma^A{}_B & \emptyset\\ \emptyset & -\bar{\Sigma}_{A'}{}^{B'}\end{pmatrix} = -\frac{1}{8}[\gamma_\alpha,\gamma_\beta]e^\alpha\wedge e^\beta.$$

- *On a null surface $\mathscr{N}$, there always exists a spinor $\ell^A : \mathscr{N}\to\mathbb{C}^2$ and a spinor-valued two-form $\eta^A{}_{ab}\in\Omega^2(\mathscr{N}:\mathbb{C}^2)$ such that*

$$\varphi^*_{\mathscr{N}}\Sigma_{ABab} = \ell_{(A}\eta_{B)ab}.$$

- The Lorentz invariant spin $(0,0)$ scalar $\boldsymbol{\varepsilon} = -\mathrm{i}\eta_A\ell^A$ defines the *oriented* area of any two-dimensional cross section $\mathscr{C}$ of $\mathscr{N}$

$$\mathrm{Area}[\mathscr{C}] = \int_{\mathscr{C}}\boldsymbol{\varepsilon} = -\mathrm{i}\int_{\mathscr{C}}\eta_A\ell^A.$$

Boundary term on the light front

Bulk plus boundary action.

- Tetradic Hilbert – Palatini action in the bulk,

$$S_{\mathscr{M}}[A,e] = \left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{M}} \Sigma_{AB}[e]\wedge F^{AB}[A]\right] + \mathrm{cc.}$$

- $SL(2,\mathbb{C})$-invariant boundary action,

$$S_{\mathscr{N}}[A|\eta,\ell|g] = \left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{N}} \underbrace{\eta_A\wedge\Big(D-\frac{1}{2}\varkappa\Big)\ell^A}_{\text{“pdq”}}\right] + \mathrm{cc.}$$

The boundary intrinsic one-form $\varkappa_a$ determines the non-affinity of ℓ^a.

- bulk plus boundary action

$$S[A,e|\eta,\ell|g] = S_{\mathscr{M}}[A,e] + S_{\mathscr{N}}[A|\eta,\ell|g]$$

- boundary conditions: $\delta[g] = \delta[\varkappa_a,\ell^a,m_a]/_{\sim} = 0$.

Boundary conditions

Boundary data: null generators, non-affinity, co-dyads

$$\ell^a \in V\mathscr{N}, \quad \ell^a m_a = 0, \quad \ell^b \nabla_b \ell^a = \ell^b \varkappa_b \ell^a, \quad q_{ab} = 2m_{(a}\bar{m}_{b)}.$$

Boundary conditions: $\delta[\varkappa_a, \ell^a, m_a]/_\sim = 0$

- vertical diffeomorphisms $[\varphi^* \varkappa_a, \ell^a, \varphi^* m_a] \sim [\varkappa_a, \varphi_* \ell^a, m_a]$
- dilations $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \nabla_a f, \mathrm{e}^f \ell^a, m_a]$
- complexified conformal transformations $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a, \mathrm{e}^{\frac{1}{2}(\lambda+\bar{\lambda})} \ell^a, \mathrm{e}^{\lambda} m_a]$
- shifts $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \bar{\zeta} m_a + \zeta \bar{m}_a, \ell^a, m_a]$

The equivalence class $g = [\varkappa_a, \ell^a, m_a]/_\sim$ characterises two degrees of freedom per point.

Role of the Barbero – Immirzi parameter

Boundary term for the Holst action

Bulk plus boundary action.

- Holst action in the bulk,

$$S_{\mathscr{M}}[A,e]=\frac{\gamma+\mathrm{i}}{\gamma}\left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{M}}\Sigma_{AB}[e]\wedge F^{AB}[A]\right]+\mathrm{cc}.$$

- $SL(2,\mathbb{C})$-invariant boundary action,

$$S_{\mathscr{N}}[A|\eta,\ell|g]=\frac{\gamma+\mathrm{i}}{\gamma}\left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{N}}\underbrace{\eta_A\wedge\Big(D-\frac{1}{2}\varkappa\Big)\ell^A}_{\text{“pdq”}}\right]+\mathrm{cc}.$$

  The one-form $\varkappa_a\in\Omega^1(\mathscr{N})$ is the null surface analogue of the Ashtekar – Barbero connection

- bulk plus boundary action

$$S[A,e|\eta,\ell|g]=S_{\mathscr{M}}[A,e]+S_{\mathscr{N}}[A|\eta,\ell|g]$$

- boundary conditions: $\delta[g]=\delta[\varkappa_a,\ell^a,m_a]/_{\sim}=0$.

Immirzi parameter twists the boundary conditions

Complex abelian connection for $U(1)\times \mathrm{dilations}.$

$$\ell^a D_a \ell^A = \frac{1}{2}\Big(\kappa_{(\ell)} + \mathrm{i}\varphi_{(\ell)}\Big)\ell^A.$$

Boundary connection: sum of 'extrinsic curvature' and 'spin connection'.

$$\ell^a \varkappa_a = \kappa_{(\ell)} - \gamma^{-1}\varphi_{(\ell)}.$$

Boundary conditions: $\delta[\varkappa_a, \ell^a, m_a]/_\sim = 0$

- vertical diffeomorphisms $[\varphi^*\varkappa_a, \ell^a, \varphi^* m_a] \sim [\varkappa_a, \varphi_*\ell^a, m_a]$
- dilations $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \nabla_a f, \mathrm{e}^f \ell^a, m_a]$
- complexified conformal transformations $\lambda = \mu + \mathrm{i}\nu$:
  $[\varkappa_a, \ell^a, m_a] \sim \Big[\varkappa_a - \frac{1}{\gamma}\nabla_a \nu, \mathrm{e}^{\mu}\ell^a, \mathrm{e}^{\mu+\mathrm{i}\nu} m_a\Big]$
- shifts $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \bar{\zeta} m_a + \zeta \bar{m}_a, \ell^a, m_a]$

The equivalence class $g = [\varkappa_a, \ell^a, m_a]/_\sim$ characterises two degrees of freedom per point.

Graviton and $SL(2,\mathbb{R})$

The two degrees of freedom can be neatly organised into an $SL(2,\mathbb{R})$ element.

Define auxiliary $SL(2,\mathbb{R}) \ni S$ holonomy

$$\ell^a \partial_a S = \big(\varphi_{(\ell)} J + \sigma_{(\ell)} \bar{X} + \bar{\sigma}_{(\ell)} X\big) S,$$

where $(J, X, \bar{X})$ are generators of $SL(2,\mathbb{R})$

$$\begin{aligned}[J, X] &= -2\mathrm{i}X,\\ [X, \bar{X}] &= +\mathrm{i}J.\end{aligned}$$

Two physical degrees of freedom encoded into homogenous space $SL(2,\mathbb{R})/U(1)$ modulo vertical diffeomorphisms.

From quasi-local phase space to radiative phase space at infinity

Symplectic potential on null surface boundary

We send $\gamma \to \infty$.

- Covariant pre-symplectic potential along the portion of the null surface between $\mathscr{C}_0$ and $\mathscr{C}_1$

$$\Theta_{\mathscr{N}} = -\frac{1}{8\pi G}\int_{\mathscr{N}} \big(\varepsilon \wedge \mathbb{d}\varkappa - k_a \mathbb{d}\ell^a \mathrm{d}\varepsilon + \frac{1}{2}\vartheta_{(\ell)} k \wedge \mathbb{d}\varepsilon\big) + \\ + \frac{\mathrm{i}}{8\pi G}\int_{\mathscr{N}} \big(\sigma_{(\ell)} k \wedge \bar{m} \wedge \mathbb{d}\bar{m} - \mathrm{cc.}\big).$$

- If $\mathscr{N}$ is an isolated horizon $\mathscr{N} = \Delta$ this simplifies to

$$\Theta^{\mathrm{IH}}_{\Delta} = \frac{1}{8\pi G}\int_{\Delta} \varepsilon \wedge \mathbb{d}(k_a \nabla \ell^a).$$

- On the other hand, in the limit $\mathscr{N} \to \mathscr{I}^+$ to null infinity, we recover the symplectic structure for the two radiative modes,

$$\Omega^{\mathrm{rad}}_{\mathscr{I}^+}(\delta_1, \delta_2) = \frac{1}{4\pi G}\int_{\mathscr{I}^+} \mathrm{d}u \wedge \boldsymbol{\varepsilon}\big(\delta_{[1}\dot{\sigma}\,\bar{\delta}_{2]}\sigma + \mathrm{cc.}\big).$$

Corner term in the symplectic potential on partial Cauchy surfaces

Covariant pre-symplectic potential for the partial Cauchy surfaces:

$$\Theta_\Sigma = \left[-\frac{\mathrm{i}}{8\pi G} \oint_{\mathscr{C}} \eta_A \mathbb{d}\ell^A + \frac{\mathrm{i}}{8\pi G} \int_\Sigma \Sigma_{AB} \wedge \mathbb{d}A^{AB} \right] + \mathrm{cc}.$$

Gauge symmetries:

- Simultaneous $SL(2,\mathbb{C})$ transformations of bulk plus boundary fields.
- Small diffeomorphisms that vanish at the corner $\xi^a\big|_{\mathscr{C}} = 0$.
- $U(1)$ transformations of the boundary spinors.

Quasi local observables

- Tangential diffeomorphisms (for $\xi^a\big|_{\mathscr{C}} \in T\mathscr{C}$) are integrable

$$\Omega_\Sigma(\mathscr{L}_\xi, \delta) = -\delta J_\xi[\mathscr{C}], \quad \text{for:} \quad \begin{cases} \mathscr{L}_\xi A^A{}_B = \xi \lrcorner F^A{}_B\,, \\ \mathscr{L}_\xi \Sigma_{AB} = \xi \lrcorner \nabla(\Sigma_{AB}) + \nabla(\xi \lrcorner \Sigma_{AB}). \end{cases}$$

- Dilatations of the boundary spinors are integrable

$$\Omega_\Sigma(\delta_\lambda, \delta) = -\delta K_\lambda[\mathscr{C}], \quad \text{for:} \quad \begin{cases} \delta_\lambda \ell^A = +\frac{\lambda}{2}\ell^A, \\ \delta_\lambda \eta_{Aab} = -\frac{\lambda}{2}\eta_{Aab}. \end{cases}$$

Quasi local observables

diffeomorphisms: $J_\xi[\mathscr{C}] = \dfrac{\mathrm{i}}{8\pi G}\displaystyle\int_{\mathscr{C}} \Big[\eta_A \mathscr{L}_\xi \ell^A - \mathrm{cc.}\Big]$, for all $\xi^a\big|_{\mathscr{C}} \in T\mathscr{C}$.

dilatations: $K_\lambda[\mathscr{C}] = -\dfrac{\mathrm{i}}{16\pi G}\displaystyle\int_{\mathscr{C}} \lambda\Big[\eta_A \ell^A - \mathrm{cc.}\Big] = \dfrac{1}{8\pi G}\displaystyle\int_{\mathscr{C}} \lambda \boldsymbol{\varepsilon}.$

Role of Bondi energy

- Supertranslations $\xi^a \in [\ell^a]$ generated by time-dependent Hamiltonian

$$\delta\big[H_\xi[\partial\Sigma]\big] = -\Omega_\Sigma(\mathscr{L}_\xi, \delta) + \oint_{\partial\Sigma} \xi \lrcorner \theta_{\mathscr{N}}(\delta).$$

- Corner term results from time-dependence.
- Recall toy model with time-dependent parameters $\omega_I(t)$.
- If $\delta \in T\mathscr{H}_{phys}$ is a tangent vector to the space of physical histories,

$$\delta[H]\big|_{\mathscr{H}_{phys}} = -\Omega_S\Big(\frac{\mathrm{d}}{\mathrm{dt}}, \delta\Big)\Big|_{\mathscr{H}_{phys}} + \frac{\partial H_{int}[p_i, q^i|\omega_I(t)]}{\partial \omega_I}\delta[\omega_I(t)]\Big|_{\mathscr{H}_{phys}}.$$

Limit to $\mathscr{I}^+$ for $\xi^a = \frac{r}{2}\vartheta_{(k)}\ell^a$ returns Bondi mass = *free energy*

Bondi energy: $M_{\mathrm{B}}(u) = -\dfrac{1}{4\pi G}\displaystyle\oint_{\mathscr{C}_u} d^2\Omega \left(\bar{\Psi}_2^{(0)} + \sigma^0 \dot{\bar{\sigma}}^0 - \bar{\mathscr{D}}^2 \bar{\sigma}^0\right),$

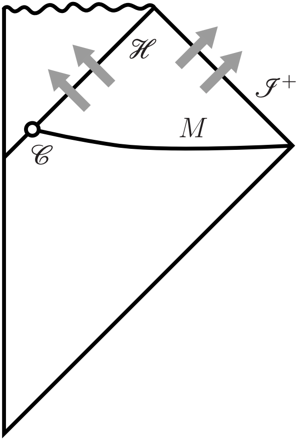
Free energy: $\displaystyle\lim_{\rho\to\infty} \delta\big[H_\xi[\mathscr{C}_{\rho,u}]\big] = -\frac{1}{8\pi G}\delta\Big[\oint_{\mathscr{C}_u} \varepsilon\,\kappa\Big] + \delta\big[M_{\mathrm{B}}(u)\big].$

Conclusion and Summary

Edge modes vs. radiative modes

A boundary breaks the gauge symmetries in the bulk and turns them into physical boundary modes (boundary gravitons, edge modes, pseudo Goldstone boson ...).

Physical phase space: $\mathscr{P}_M = [\mathscr{P}_M^{\text{bulk}} \times \mathscr{P}_{\partial M}^{\text{boundary}}]/_{\text{gauge}}$



- In spacetime dimensions $d < 4$, there are no degrees of freedom in the bulk. Physical phase space is the phase space of boundary field theory alone.
- Treat gravity as a time dependent Hamiltonian system. Remove the radiative modes from the Cauchy hypersurface M. Encode them into auxiliary background fields. Probably enough to understand BH entropy at the full non-perturbative level.

$$\Omega_M(\delta, L_\xi) = \delta M - \Omega \delta J - \kappa \delta A = 0.$$