Panel on Boundary Modes and Celestial Holography

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ILQGS

03-05-2022

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Corners break symmetries and turn otherwise unphysical gauge directions into physical boundary modes.

Origin of intriguing developments in our field.

At the corner, symmetries are broken: corner conflict in classical architecture.

- complementary representation of LQG quanta of area.
- quasi-local observables in no-perturbative quantum gravity.
- gluing and coarse graining.
- towards radiative modes and scattering in LQG.
- Wheeler De Witt equation on null surface = Ward identities.
- Quantum reference frames.
- BH entropy/area counting.

Pre-symplectic potential for first-order gravity*

$$\Theta_\Sigma = \frac{1}{16\pi G}\Big[\int_\Sigma *(e_\alpha \wedge e_\beta)\wedge \mathbb{d}A^{\alpha\beta} + 2\oint_{\partial\Sigma} *(e_\alpha\wedge e_\beta) n^\alpha \mathbb{d} n^\beta\Big].$$

Are the radiative modes the only Dirac observables that we have?

To know the dimension of a manifold it is enough to know the dimension of its tangent space.

Warm up: Take linearised gravity in first-order variables

$$e^\alpha = \Lambda^\alpha{}_\mu\big(\mathrm{d}X^\mu + f^\mu\big),$$
$$A^\alpha{}_\beta = \Lambda^\alpha{}_\mu \mathrm{d}\Lambda_\beta{}^\mu + \Lambda^\alpha{}_\mu \Delta^\mu{}_\nu \Lambda_\beta{}^\nu.$$

Evaluate Ω_Σ in a round ball centred at origin in X^μ-space.

* see [Corichi, Wilson-Ewing 2011; WW 2011, WW 2017; Freidel, Geiller, Pranzetti 2020; Bodendorfer, Neiman 2013; Margalef-Bentabol, Villaseñor, Barbero G. 2021; ...]

Fluctuations around Minkowski space

Only at the corner, do X^μ-fluctuations become physical

$$\Omega_\Sigma\Big|_{Minkowski} = \frac{1}{8\pi G}\int_\Sigma \mathrm{d}X_{[\mu} \wedge \mathbb{d} f_{\nu]} \curlywedge \mathbb{d}(*\Delta^{\mu\nu}) + \\ - \frac{1}{8\pi G}\oint_{\partial\Sigma}\Big[\underbrace{\mathbb{d}(*F_{\mu\nu})\curlywedge X^{[\mu}\mathbb{d}X^{\nu]}}_{\longrightarrow\ ADM\ charges\ at\ spi^*} + \\ + d^2v\,\rho\,\varepsilon^{ab}\varepsilon^{cd}\mathbb{d}K_{ac}\curlywedge D_{(b}\mathbb{X}^{\downarrow}_{d)} + \\ + d^2v\,\rho\,\varepsilon^{ab}\varepsilon^{cd}\mathbb{g}_{ac}\curlywedge D_b D_d \mathbb{T}, \\ - \frac{1}{2} d^2v^a\big(\mathbb{g}_{ab} - h_{ab}\mathbb{g}^d{}_d\big)\curlywedge \mathbb{N}^b\Big].$$

3+1 split of basic variations

$$\partial^a_\mu \mathbb{d}X^\mu \equiv \mathbb{X}^a = \mathbb{T}n^a + \mathbb{X}^a_\downarrow.$$

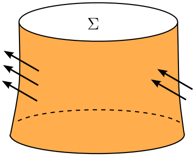
* [Ashtekar, Hansen 1978]

Radiation and charge

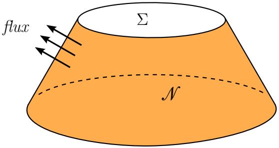
In linearised gravity, diffeo charges are trivially integrable (as in 2+1). Not so in full non-linear theory (because of radiation). No canonical generators for time-like (or radial) diffeos. Subsystems characterised by charges, flux of radiation and choice of boundary embedding.

- Choice for how to extend the corner $\partial\Sigma$ into a worldtube $\mathscr{N}$.
- Choice for what is the flux of radiative modes crossing the boundary.
- Most ingenious idea [Freidel, Barnich, Troessaert, Leigh, Ciambelli], see also [Riello, Gomes], to study Komar charges on phase space via field-space connection. More investigation needed.

$$\Omega^{new}_{\Sigma} = \mathbb{D}\Theta^{new}_{\Sigma}, \quad \mathbb{D} = \mathbb{d} - \mathscr{L}_{\mathbb{X}},$$
$$\Omega^{new}_{\Sigma}(\delta, \mathscr{L}_{\xi}) = \mathbb{D}_{\delta}\big[\Theta^{new}_{\Sigma}(\mathscr{L}_{\xi})\big] - \underbrace{\Theta^{new}_{\Sigma}\big(\mathscr{L}_{[\mathbb{X}(\delta),\xi]}\big)}_{flux}.$$



vs.



[Freidel, Barnich, Troessart, Leigh, Ciambelli, Riello, Gomes, Speranza, Chen, Donnay, Chandrasekaran, ww, ...]

Bulk plus boundary action:

$$S = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\left[\int_{bulk} \Sigma_{AB} \wedge F^{AB} + \int_{null\text{-}boundary} \eta_A \wedge \left(D - \tfrac{1}{2}\varkappa\right)\ell^A\right] + \mathrm{cc.}$$

Boundary conditions along $\mathscr{N}$: $\delta[\varkappa_a, l^a, m_a]/_\sim = 0.$

Covariant pre-symplectic potential for the partial Cauchy surfaces:

$$\Theta_\Sigma = \frac{\mathrm{i}}{8\pi\gamma G}(\gamma + \mathrm{i})\left[\int_{disk} \Sigma_{AB} \wedge \mathbb{d}A^{AB} - \oint_{corner} \eta_A \mathbb{d}\ell^A\right] + \mathrm{cc.}$$

Heisenberg algebra at the two-dimensional corner

$$\{\eta_{Aab}(z), \ell^B(z')\}_{\mathscr{C}} = -\frac{8\pi\mathrm{i}\gamma G}{\gamma + \mathrm{i}}\delta^B_A \tilde{\delta}^{(2)}(z, z')\underset{\sim}{\varepsilon}_{ab}.$$

$$\widehat{\text{Area-flux}}[\mathscr{C}]\Psi_{\text{phys}} = 4\pi\gamma\hbar G/c^3 \oint_{\mathscr{C}} \left[a^\dagger_A a^A - b^\dagger_A b^A\right]\Psi_{\text{phys}}.$$

Signature $(0{+}{+})$ metric.

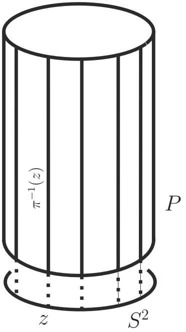
$$q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i,j = 1,2.$$

Parametrisation of the dyad

$$\boxed{e^i = \Omega\, S^i{}_j\, e^j_{(o)}.}$$

Choice of time:

$$\partial_U^b \nabla_b \partial_U^a = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}U}\Omega^2)\partial_U^a$$



Kinematical phase space for radiation: $\mathscr{P}_{kin} = \mathscr{P}_{abelian} \times T^* SL(2,\mathbb{R})$.

$$\Theta_{\mathcal{N}} = \frac{1}{8\pi G} \int_{\mathcal{N}} d^2 v_o \wedge \left[p_K \, \mathbb{d}\widetilde{K} + \frac{1}{\gamma} \Omega^2 \, \mathbb{d}\widetilde{\Phi} + \widetilde{\Pi}^i{}_j \left[S \mathbb{d} S^{-1} \right]^j{}_i \right] + \textit{corner term}.$$

Abelian variables:

$U(1)$ connection: $\widetilde{\Phi}$, area: $\Omega^2 \, d^2 v_o$, lapse: $\widetilde{K} := \text{đ}U$, expansion: p_K.

Upon imposing 2nd-class constraints: Dirac bracket for radiative modes

$$\left\{ S^i{}_m(x), S^j{}_n(y) \right\}^* = -4\pi G \, \Theta(U_x, U_y) \, \delta^{(2)}(\vec{x}, \vec{y}) \, \Omega^{-1}(x) \, \Omega^{-1}(y) \times \left[\mathrm{e}^{-2\,\mathrm{i}\,(\Delta(x) - \Delta(y))} \left[X S(x) \right]^i{}_m \left[\bar{X} S(y) \right]^j{}_n + \mathrm{cc.} \right].$$

Gauge symmetries:

1. $U(1)$ transformations
2. vertical diffeomorphisms along null generators

Summary

Main results discussed:

- Boundary conditions for radiative data altered by Barbero–Immirzi parameter.
- Barbero–Immirzi parameter mixes $U(1)$ frame rotations and dilations. **This is an important observation – it is the geometric origin for LQG quantum discreteness of area.**
- Poisson brackets for the boundary modes altered by addition of the Immirzi parameter. Poisson brackets for radiative modes unchanged.
- New representation of quantum geometry.

Future of the programme:

- Ward identities/constraints linking corner data to flux.
- State-amplitude correspondence as we know it from spinfoams: physical states: $\Psi_{\mathscr{N}}(\text{corner data}_+|\text{radiative data}|\text{corner data}_-)$.

$$\langle\text{out}|\text{in}\rangle_{\text{EPRL-foam?}} = \sum_C \bar{\Psi}_{\mathscr{N}^-}(\text{c}_{+\infty}|\text{out}|C)\Psi_{\mathscr{N}^+}(C|\text{in}|\text{c}_{-\infty}).$$

- Impulsive waves and spinfoams, building upon [ww2016].